Yevgenii SHTEFAN,
Doctor of Technical Sciences, Full Professor,
National Technical University of Ukraine
"Igor Sikorsky Kyiv Polytechnic Institute"
ORCID ID: 0000-0002-0697-7651

Anatoliy MIKHAILOV,
Doctor of Philosophy of Mechanical Engineering
Frantsevich Institute for Problems of Materials Science
National Academy of Science of Ukraine
ORCID ID: 0000-0003-1442-0658

Oleg MIKHAILOV,
Doctor of Technical Science, Leading Researcher
Frantsevich Institute for Problems of Materials Science
National Academy of Sciences of Ukraine
ORCID ID: 0000-0001-6040-2376

MODELING OF FORMING PROCESSES OF POWDER AND POROUS MATERIALS PRODUCTS

Introduction

Successful development of mechanical engineering is possible on the basis of the introduction of effective resource-saving technologies. Such technologies include powder metallurgy methods. Powder metallurgy methods have a number of advantages that allow them to replace more expensive metal processing methods: casting, forging and stamping, as well as mechanical processing. Recent years have been characterized by a significant expansion of the application of processes for deforming powder materials and porous blanks. The use of porous blanks allows them to be compacted and give the product a given shape with less effort, reduce the number of transitions and reduce the number of required stamping tools.

When designing powder metallurgy technological processes, it is relevant to study the influence of deformation schemes on the distribution of parameters that determine the characteristics of products. Along with traditional methods, information technologies and computer modelling methods are currently increasingly used in production design. The development of effective modelling methods is an important problem.

There are two main approaches used in modelling the deformation processes of powder materials and porous blanks: discrete and continuous. The discrete approach considers the interaction between individual elements (particles) of a dispersed medium[1]. It is implemented in the discrete element method. Discrete element modelling allows us to obtain information about the trajectories of individual particles and the forces acting on them, which, in turn, allows us to better understand the deformation processes. The discrete element method does not have the disadvantages of the continuous approach associated with the violation of the continuity of the deforming body; it allows us to model the behaviour of multicomponent media. The disadvantages of discrete methods include high computational costs, the complexity of determining the characteristics necessary for the design of technological processes, and limited application. Therefore, the main approach in modelling the deformation processing processes of powder materials is the continuous approach. Introducing continuous representations, we proceed from the fact that the behaviour of an individual powder particle does not represent the behaviour of the entire powder mass. Unlike individual powder particles, the plastic deformation of which occurs without a change in volume, the external load of the entire set of particles is accompanied by significant irreversible volume changes, i.e., powder and porous bodies can be considered as a medium that is plastically compressed. The smallest volume of the ensemble in which the powder particles are in contact with each other and reproduces the behaviour of the entire powder mass is called a representative element. Its volume is usually several orders of magnitude smaller than the volume of the entire powder mass. It is assumed that the entire powder volume can be composed of representative elements. Local characteristics of the powder medium, such as density, temperature, stress tensors, displacement velocities, and others, are introduced in accordance with the canons of a continuous medium with the caveat that a point of the powder medium does not mean a separate particle of powder, but a representative element. The most common

1 Martin C. L., Bouvard D., Delette G. Discrete element simulations of the compaction of aggregated ceramic powders. Journal of the American Ceramic Society. 2006. Vol. 89, Num. 11, P. 3379-3387

modelling method for solving problems of deformation of powder and porous materials is the finite element method, which is implemented in a number of universal software systems. Models of powder and porous bodies are implemented in modern programs, which makes it possible to model the deformation processes of dispersed media. However, different plastic flow models are used for powders and porous bodies, which cause a number of inconveniences in modelling. In addition, not all programs implement these models simultaneously.

Research Methodology

A method for studying the regularities of the processes of forming parts from powder and porous materials has been developed. The relations of the theory of plasticity of a porous body[2] and the finite element method are used. The scientific substantiation of resource-saving technologies for manufacturing machine parts manufacturing is proposed by the mathematical model of structurally inhomogeneous porous materials forming processes. The analytical component of the model (analytical model) is represented by a closed mathematical relations system of mathematical physics boundary value problem type. Analytical model describes the deformation process of dispersed (powder or porous structure) materials. The equation of the contour of the load surface in the p – plane (Fig. 1) has the form:

$$F = \frac{(p - p_0)^2}{\psi} + \frac{\tau^2}{\varphi} - \tau_s{}^2 = 0 \tag{1}$$

where p is the magnitude of the hydrostatic pressure; τ is the intensity of the tangential stresses; φ and ψ are material functions depending on porosity; p 0 is the value of the hydrostatic component of the stress state at which the volume of the porous material does not change; τ_s is the yield strength of the powder material or the solid phase of the porous body. The expressions for the material functions are defined as follows:

$$\psi_0 = \frac{2}{3} \cdot \frac{(1-\theta)^3}{\theta} \tag{2}$$

$$\varphi = \frac{1}{(1+m)^2} \cdot (1-\theta)^3 \cdot \left(1 - |2 \cdot a - 1|\right)^2 \tag{3}$$

2 M. B. Shtern, O. V. Mikhailov, A. O. Mikhailov, Theory and technology of forming process generalized continuum model of plasticity of powder and porous materials, Powder Metallurgy and Metal Ceramics, Vol. 60, Nos. 1-2, May, 2021 (Russian Original Vol. 60, Nos. 1-2, Jan.-Feb., 2021. – pp. 27-44). – pp.20-36 DOI: https://doi.org/10.1007/s11106-021-00211-7

$$\psi_1 = 4(1-\theta)\psi_0 \cdot \frac{(1-a)^2}{(1+m)^2} \tag{4}$$

$$\psi_2 = 4(1-\theta)\psi_0 \cdot \frac{a^2}{(1+m)^2} \tag{5}$$

$$p_0 = \tau_s \sqrt{1-\theta}\sqrt{\psi_0} \cdot \left(\frac{1-m-2\cdot a}{1+m} \right) \tag{6}$$

$$\psi = \frac{1}{2}(1 - Sign(p-p_0)\ \psi_1 + \frac{1}{2}(1+Sign(p-p_0)\ \psi_2 \tag{7}$$

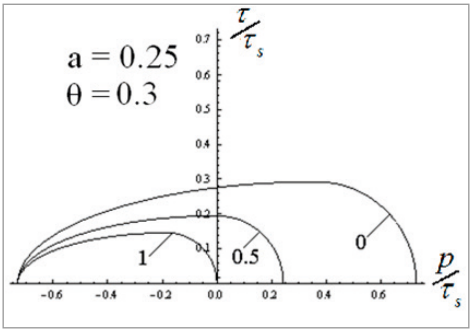


Figure 1. Appearance of the load surface contour in the p – plane for different values of the parameter m

The general relationship between stresses and strain rates in component-wise form is determined by the flow law associated with the plastic potential equation in the form[3]:

$$\sigma_{ij} = \frac{\tau_s \cdot \varphi}{w}\left[e_j + \left(\frac{\psi}{\varphi} - \frac{1}{3} \right) \cdot e \cdot \delta_j \right] + p_0 \cdot \delta_j \tag{8}$$

From the expression for the specific energy dissipation rate and the hypothesis that this energy is dissipated only in connection with plastic deformation, a kinetic equation for strain accumulation follows. The strain hardening of the solid phase can be taken into account using equation:

$$w = \frac{\sqrt{\gamma^2\varphi + e^2\psi}}{\sqrt{1-\theta}} + \frac{p_0}{\tau_s} \cdot \frac{e}{1-\theta} \tag{9}$$

3 Shtefan, E., Pashchenko, B., Blagenko, S., Yastreba, S. (2019). Constitutive Equation for Numerical Simulation of Elastic-Viscous – Plastic Disperse Materials Deformation Process. In: Ivanov, V., et al. Advances in Design, Simulation and Manufacturing. DSMIE 2019. Lecture Notes in Mechanical Engineering. Springer, Cham. DOI: https://doi.org/10.1007/978-3-319-93587-4_37

The adopted rheological model of the material allows describing the deformation of both powder and porous workpieces. It takes into account the different resistance of these materials under tension and compression.

The numerical implementation of the problems of modelling metal processing processes by pressure uses one of the most well-known variation principles of mechanics – the principle of virtual velocities. According to this principle, for any kinematic possible velocity field at an arbitrary moment of time, the strengths of internal and external forces are equal:

$$\int_V \sigma_{ij} e_{ij} dV = \int_S p_i v_i dS, \tag{10}$$

Equation (10) is called the equation of virtual powers. It means the equality of the powers of internal forces (stresses $\boldsymbol{\sigma}$ij) in the volume of the body V in an arbitrary (virtual) velocity field and the power of external forces in the same velocity field. Based on the variation principle, the real velocity field is the result of finding the functional extreme, which has the form:

$$J = \int_V \sigma_{ij} e_{ij} dV - \int_S p_i v_i dS \tag{11}$$

Taking into account the determining relations of the plastic flow of powder and porous materials, equation (11) will take the form:

$$J = \int_V \tau_s \left(\frac{\sqrt{\gamma^2 \varphi + e^2 \psi}}{\sqrt{1-\theta}} + \frac{p_0}{\tau_s} \cdot \frac{e}{1-\theta} \right) dV - \int_S p_i \cdot v_i dS, \tag{12}$$

where *pi* – loads applied to the surface of the deforming body; v_i – velocities of movement of points of the body deforming under the action of loads p_i; V – volume of the body; S – surface of the body. The system of differential equations obtained as a result of finding the extreme of the functional (12) is replaced by a discrete finite element model, which is a system of nonlinear algebraic equations:

$$[K] \cdot \left\{ \dot{X} \right\} = \{P\} - \{P_0\}, \tag{13}$$

where $[K] = \sum_{1}^{Ne} \int_V [B]^T_{(e)} [D]_{(e)} [B]_{(e)} dV$ –

global stiffness matrix of the system; N_e – number of finite elements; V – element's volume; $\{\dot{X}\}$ – knot speed column; $\{P\}$ – nodal force column;

$\{P_0\} = \sum_{1}^{Ne} [B]^T_{(e)} \{P_0\}_{(e)} dV$ column of nodal forces corresponding to the magnitude of the displacement of the centre of the ellipses approximating the contour of the load surface along the axis *p* (Fig. 1); $[B]_{(e)}$ – matrix relating the rates of plastic deformation in an element to the rates of displacement of its nodes; $[D]_{(e)}$ – a matrix that relates the stress components in an element to the strain rates according (8).

The system of equations (13) is a system of nonlinear algebraic equations with respect to $\{\dot{X}\}$. At the same time, the global stiffness matrix depends on the coordinates of the nodes of the finite elements. Thus, this system of equations is also a system of differential (time) equations with respect to $\{\dot{X}\}$. The differential equations system is solved by the finite difference method. On the time interval $\Delta t = t_{m+1} - t_m$, the coordinates of the nodes of the finite elements are determined:

$$\{X\}_{m+1} = \{X\}_m + \int_{t_m}^{t_{m+1}} \left\{ [K]^{-1} (\{P\} - \{P_0\}) \right\} \cdot dt \tag{14}$$

With a two-layer approximation we obtain:

$$\begin{aligned} \Delta\{X\}_{m+1} &= (1-\alpha)\cdot \Delta t \cdot [K]^{-1}_m (\{P\} - \{P_0\})_m + \\ &+ \alpha \cdot \Delta t \cdot [K]^{-1}_{m+1} (\{P\} - \{P_0\})_{m+1} \end{aligned} \tag{15}$$

де $0 \le \alpha \le 1$, in the calculations it was considered $\alpha = 0{,}5$.

In elastic-plastic deformation of powder and porous materials, the increment of the total deformation consists of the sum of elastic $d\xi^e_{ij}$ and plastic $d\xi^p_{ij}$ deformations:

$$d\xi_{ij} = d\xi^e_{ij} + d\xi^{pl}_{ij} \tag{16}$$

Elastic deformations are reversible, and plastic deformations are irreversible.

The increments of elastic deformations are determined according to Hooke's law:

$$d\xi^e_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}(\sigma_{kk})\delta_{ij} \tag{17}$$

The increments of plastic deformations are determined according to the law associated with the flow surface (the flow occurs in the direction normal to this surface):

$$d\xi_{ij}^{pl} = \lambda \frac{\partial F}{\partial \sigma_{ij}}; \lambda > 0 \tag{18}$$

Accordingly, the increase in volumetric strain and shear strain is determined by expressions:

$$de^{pl} = \lambda \frac{\partial F}{\partial p} = \frac{2(p - p_0)}{\psi} \tag{19}$$

$$d\gamma^{pl} = \lambda \frac{\partial F}{\partial \tau} = \frac{2\tau}{\varphi} \tag{20}$$

The variation functional and the system of equations of the finite element method for the elastic-plastic deformation model take the form:

$$J = \int_V \tau_s \left(\frac{\sqrt{\gamma^2 \varphi + e^2 \psi}}{\sqrt{1-\theta}} + \frac{\dot{p}_0}{\tau_s} \cdot \frac{e}{1-\theta} \right) dV - \int_S \dot{p}_i \cdot v_i dS, \tag{21}$$

$$[K] \cdot \left\{ \dot{X} \right\} = \{\dot{P}\} - \{\dot{P}_0\} \tag{22}$$

The solution of the problems of plastic deformation of powder and porous materials was performed in stages, using the method of successive loadings. As a first approximation at each step, the solution of the problem of linear-viscous flow of the material was used. At each loading step the iterative refined method was used. The iterative process was stopped after the difference between the found values of the nodal velocities became, compared to the previous ones, smaller than a predetermined small number. Having determined the velocity field, the values of porosity, the magnitudes of the components of the stress-strain state were found and the next stage of loading was proceeded to.

For the calculation of elastic-plastic deformation process the method of successive loading was usd. At each loading step, stresses were determined, and it was assumed that the material was elastic. Then, the plastic potential F(σij) was calculated and, depending on its value, the stresses and material parameters of the model were adjusted. The algorithmic model has the form:

1. Introduction of the increment of deformations and the current values of the material parameters for the appropriated geometric model $d\xi_{ij}^{n+1}$, σ_j^n, Θ^n, ω^n

2. Determination of approximate values of stress increments: $d\sigma_{ij}^{*} = E_j^{e} d\xi_j^{n+1}$

3. Determination of approximate stresses: $\sigma_{ij} = \sigma_j^n + d\sigma_j^*$

4. Calculating the plastic potential F(σij).

If $F(\sigma ij)$ <= 0, no adjustment of stresses and material parameters is made

$$\sigma_{ij}^{n+1} = \sigma_j \,,\ \theta^{n+1} = \theta^n \,,\ \omega^{n+1} = \omega^n$$

If $F(\sigma ij) > 0$, the correction of stresses and material parameters is carried out.

4.1. Correction the increase in elastic deformations:

$$de^e = de^{n+1} - de^{pl} = de^{n+1} - \lambda \frac{\partial F}{\partial p}$$

$$d\gamma^e = d\gamma^{n+1} - d\gamma^{pl} = d\gamma^{n+1} - \lambda \frac{\partial F}{\partial \tau}$$

As a result of the iterative solution of the nonlinear equation $F(\sigma ij(\lambda)) = 0$, we determine the parameter λ.

4.2. Calculation $\sigma_j^{n+1}, \theta^{n+1}, \omega^{n+1}$

The algorithmic models were implemented in the form of corresponding digital models. The scheme of the digital model of plastic deformation is shown in Figure 2. the database using. The GID software package[4] is used as a pre- and postprocessor.

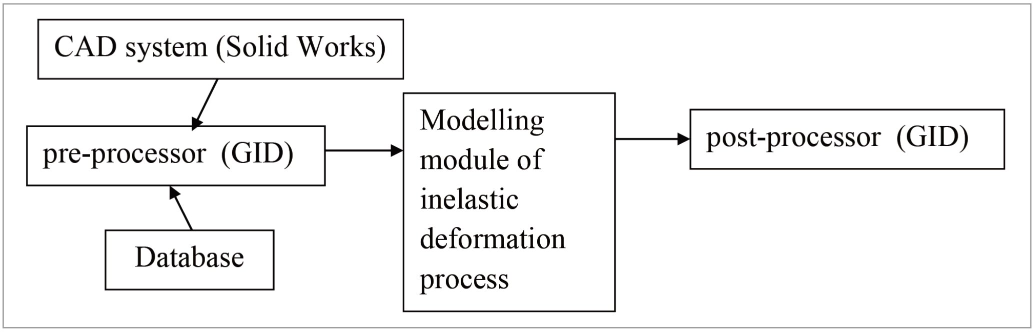


Figure 2. Scheme of the digital model of plastic deformation

The inelastic deformation problem solving module is implemented in the algorithmic programming language C. The digital elastic-plastic deformation model was implemented in the form of VUMAT and UMAT subroutines, which were implemented in the ABAQUS[5] and LS-DYNA[6] software systems,

4 GID. The personal pre and post processor developed by the International Center for Numerical Methods in Engineering (CIMNE, Barcelona, Spain). URL: https://www.gidsimulation.com

5 Abaqus. Unified FEA. (Dassault Systèmes, France). URL: https://www.3ds.com/products-services/simulia/products/abaqus/

6 LS-DYNA. (LST, USA). URL: https://www.lstc.com

respectively. VUMAT is a user-defined subroutine in ABAQUS/Explicit, written in the FORTRAN programming language. It is used for new material models specified by the user. The user can specify internal state variables that are updated together with the solution at each step. For each loading step, ABAQUS integrates the equilibrium equation based on the stress state at the beginning of the step at each integration point and passes the strain gradient tensor, stress tensor, and internal variables to the VUMAT routine. The VUMAT routine calculates the stress components and user-specified internal variables at the end of the specified strain increment for each integration point and passes them to ABAQUS. Using this information, ABAQUS can continue the calculation for the next loading step. The UMAT routine is also written in the FORTRAN programming language. Its purpose and interaction with the LS-DYNA software system are similar. A diagram showing the interaction between the VUMAT and UMAT routines and the ABAQUS and LS-DYNA software systems is shown in Figure 3.

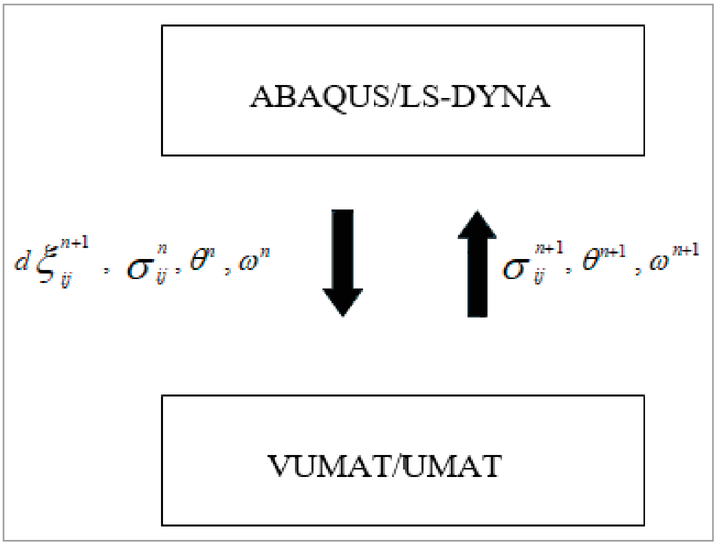


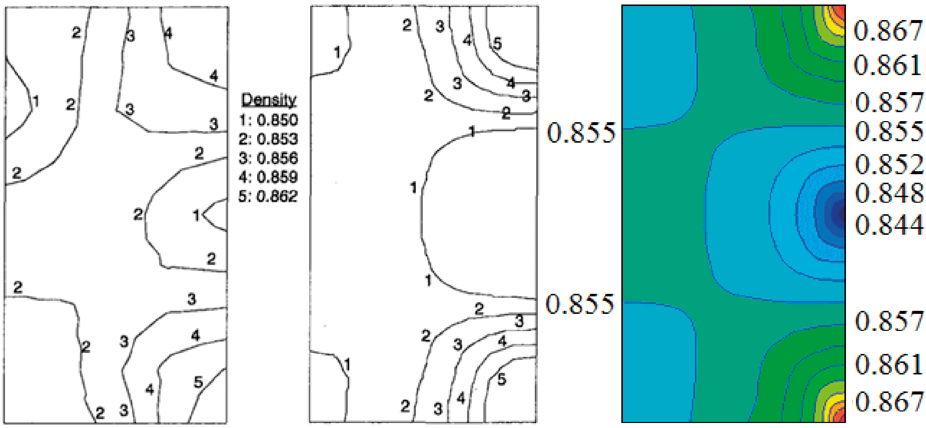
Figure 3. Interaction diagram between VUMAT and UMAT subroutines and ABAQUS and LS-DYNA software systems

Results and Discussion

The adequacy of the proposed modelling method was checked based on a comparison of the obtained results with experimental data and modelling results of other authors. The experimental data on the density distribution during the pressing of stainless steel powders are presented[7] . These processes are also simulated using the finite element method[8].

The necessary data for the computational experiment were taken from this work. The results of the calculations are shown in Figure 4. During double-sided pressing, the maximum compaction of the powder occurs in the upper and lower parts of the product. At the same time, in the middle part of the pressing, the relative density value is lower. The uneven distribution of residual porosity is due to the influence of external friction. It can also be noted that during double-sided pressing, the porosity value is distributed more evenly than during single-sided pressing.

The process of free settlement of porous cylindrical blanks was also studied. The ratio of the initial height of the blank to its diameter was equal to unity. The value of the initial porosity throughout the volume of the blank was the same and equal to 0.2. Figure 6,a shows the distribution of relative density along the axial cross-section of the blank. The modelling results show that the distribution of porosity is uneven (the figure shows a quarter of the axial cross-section of the blank).



a *б* *c*

Figure 4. Relative density distribution during double-sided pressing: experimental data[9] (a) and simulation results[10] (b) and results of computational experiment (c)

7 Kwon Y. S., Lee H. T., Kim K. T. Analysis for Cold Die Compaction of Stainless -Steel Powder. Journal of Engineering Materials and Technology, 1997. Vol. 119, Issue 4. P. 366 – 373.

8 Ibid.

9 Ibid.

10 Ibid.

Maximum compaction occurs in the central zone of the blank and at its ends near the side surface. The maximum porosity is in the area of the convexity of the side surface. At the ends of the blank in the immediate vicinity of the axis of symmetry, there is a zone of difficult deformation. In this zone, porosity is also maximum.

The metallographic studies of the surface of axial sections of deformed workpieces were carried out[11]. The shape and size of the pores were studied. In the central region of the product, maximum compaction of the material occurred. At the same time, in the zones of difficult deformation, the value of residual porosity is higher. It is also possible to note an increase in porosity in the convex region of the lateral surface of the workpiece. Thus, the modelling results correspond to the experimental data presented (Fig. 5).

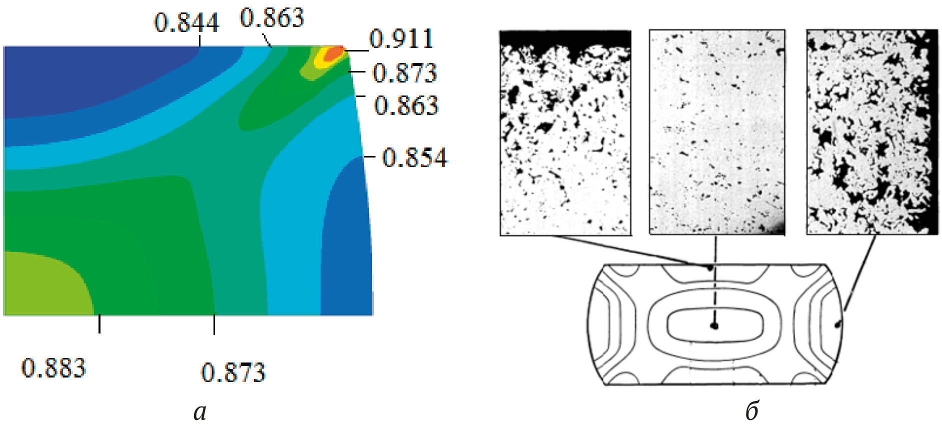


Figure 5. Relative density distribution (a) and microstructure in the cross-section of the workpiece (b)

The patterns of deformation of porous workpieces during radial compaction were studied12. The process scheme is shown in Figure 6. The workpiece is compacted by compressing it along the inner surface.

The initial porosity of the workpiece was 0.15. Its outer diameter was 120 mm, the inner diameter was 72 mm. The height of the workpiece was 85 mm. The deformation of the workpiece was carried out in two passes. Figure 7 and

11 Fischmeister H. F., Arén B., Easterling K. E. Deformation and densification of porous preforms in hot forging. Powder metallurgy. 1971. Vol. 14, Issue 27. P. 144-163.

12 A.Mykhailov, Ye. Shtefan, O. Mikhailov (2023). Modeling of manufacturing processes of thin-walled bushings from porous blanks using direct extrusion and radial compaction. USPIHI MATERIALOZNAVSTVA. № 7. KIEV: FRANTSEVICH INSTITUTE FOR PROBLEMS OF MATERIALS SCIENCE NASU,P. 19 – 26. DOI: HTTPS://DOI.ORG/10.15407/MATERIALS. URL: HTTP://WWW.MATERIALS.KIEV.UA/ISSUE/224/ARTICLE/3529

Figure 8 show the simulation results. The material compaction occurs locally (Fig. 7). First, the upper part of the workpiece is compacted. Then, according to the displacement of the punch, the compaction zone moves down. At the end of the first pass, all the material in the area of the inner surface of the workpiece is compacted. The maximum compaction of the material occurred in the centre, and to a lesser extent at the ends of the workpiece. With increasing radius, the porosity value increases and takes on a maximum value on the outer surface of the workpiece. Due to the flow of material in the vertical direction in the lower part of the workpiece, on its inner surface, a burr is formed. The diameter of the punch after the first pass was increased. In the second pass, the area of compacted material increases and extends towards the outer surface of the workpiece. However, on the outer surface of the workpiece, especially at its ends, the workpiece material is compacted slightly. The size of the burr in the lower part of the product increased. Its formation was partially compensated by the initial shape of the workpiece due to the presence of a notch in its lower part. A small burr was also formed in the area of the upper end of the product. The evolution of the accumulated plastic deformation is similar to the change in the relative density (Fig. 8). The maximum accumulated deformation is on the inner surface of the workpiece, and the minimum value is on its outer surface.

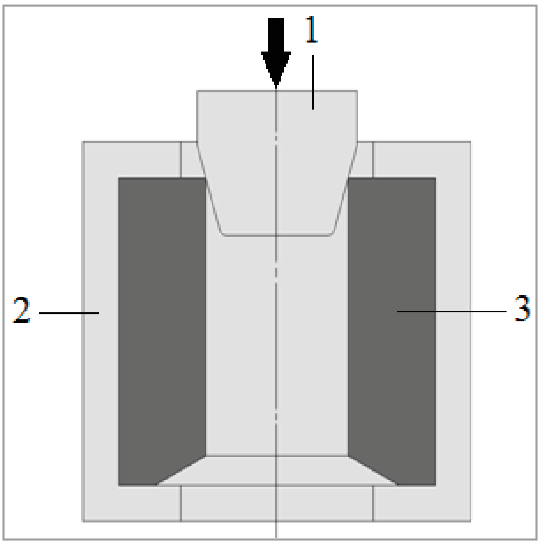
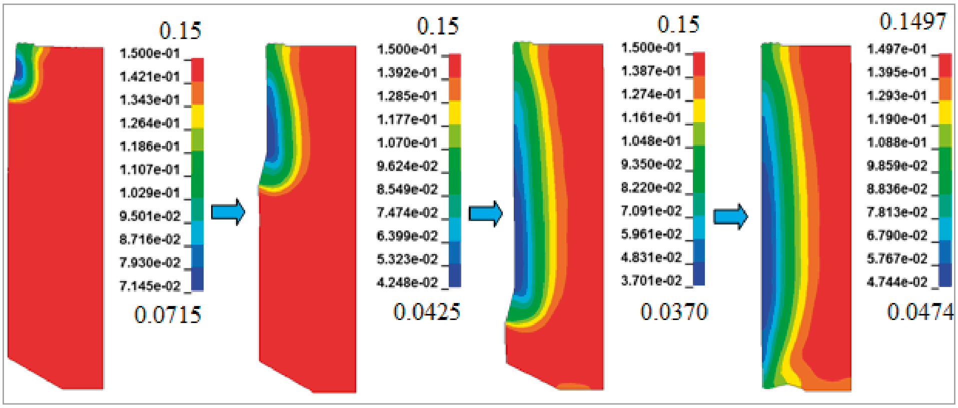


Figure 6. Calculation scheme of radial sealing:
1 – punch; 2 – matrix; 3 – workpiece



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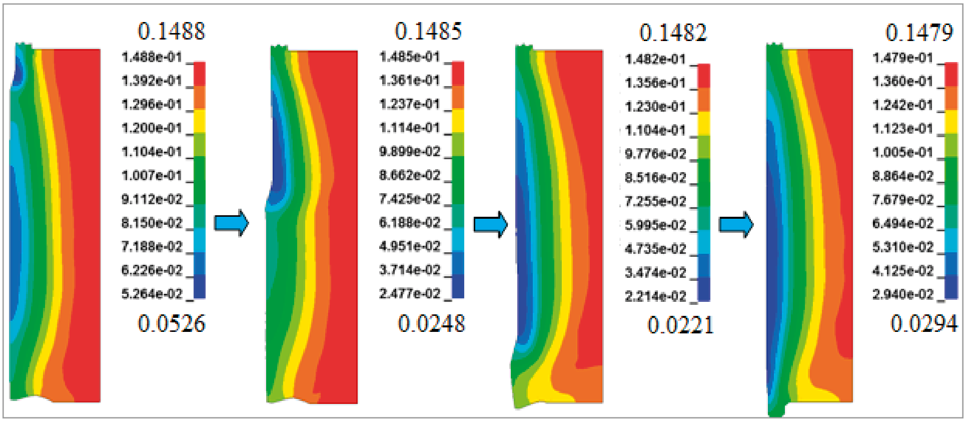

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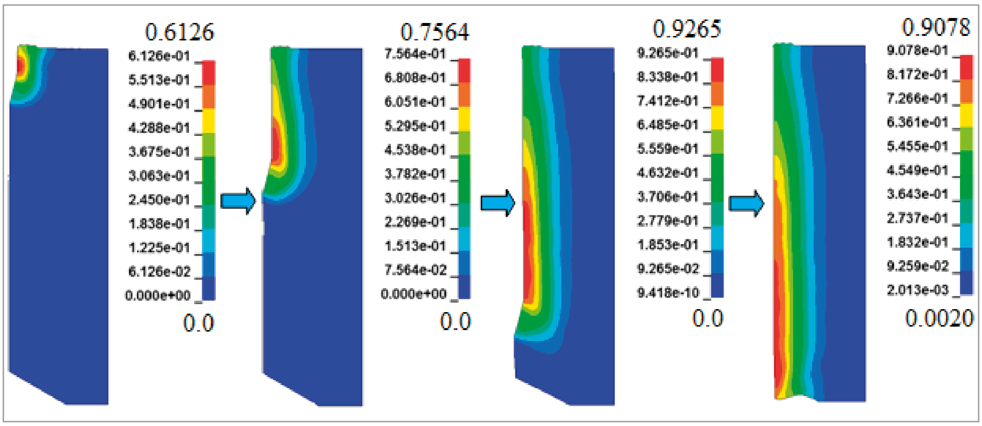
Figure 7. Analysis of the evolution of porosity fields during the first (a) and second (b) passes



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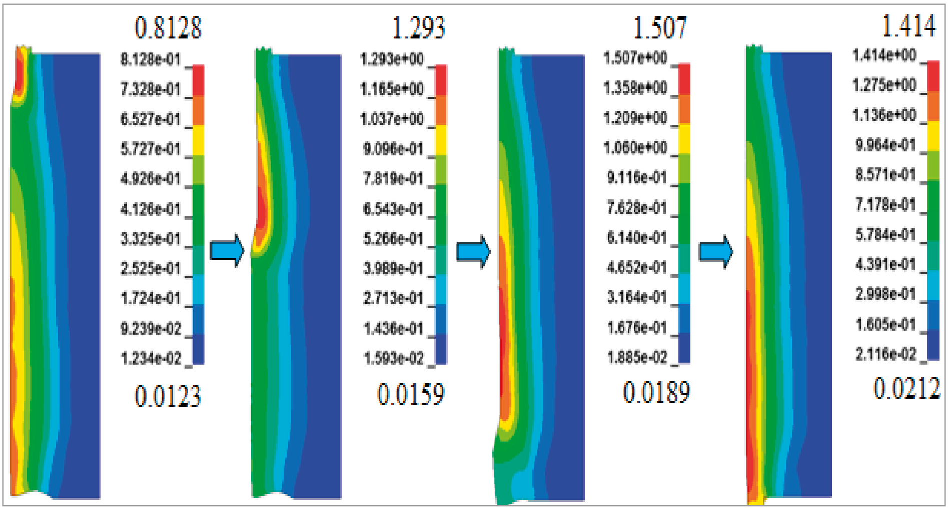

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Figure 8. Analysis of the evolution of the fields of the magnitude of the accumulated plastic deformation during the first (a) and second (b) passes

Conclusions

A mathematical model of the deformation processes of powder and porous materials has been developed, which includes analytical, algorithmic and digital models as structural components. The relations of the plasticity theory used allow us to describe the deformation of both powder and porous workpieces. The plastic and elastic-plastic behaviour of the deforming material has been considered. The results of the computational experiment correspond to the data obtained by other authors. The regularities of the deformation of porous workpieces during radial compaction along the inner diameter have been studied. It has been established that the deformation of the material occurs locally. With an increase in the radius, the porosity value along the cross-section of the product increases. Increasing the number of compressions reduces the uneven distribution of residual porosity and its magnitude. However, the uneven distribution along the radius remains.

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